

# Simulation accuracy of long range muon propagation in medium: analysis of error sources

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Knowledge of atmospheric muon flux intensity at large depths is extremely important for neutrino telescopes located deep under ground, water or ice. One of the methods to transform muon sea-level spectrum into depth one is to apply Monte Carlo technique which directly takes into account stochastic nature of energy loss. In order to decrease computation time down to acceptable level one has to use simplifications resulting in systematic errors which in some cases may distort result essentially. Here in this paper we present our analysis for dependence of computed depth muon flux upon the most important parameters of muon transport Monte Carlo algorithm which was done with the MUM (MUons+Medium) code. Contribution of different simplifications to the resulting error is considered, ranked and compared with uncertainties which come from parametrization accuracy both for sea-level muon spectrum and for muon cross sections.

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## I. INTRODUCTION

In order to obtain muon spectrum and value for integral muon flux at large depths where existing and planned neutrino telescopes are located (one to several kilometers of water equivalent) one has to perform several steps starting with interactions of primary cosmic rays in atmosphere. The last step of this procedure is transportation of muons from sea level down to detector location. It can be done both with analytical or semianalytical technique and Monte Carlo (MC) simulation which directly takes into account that muon energy loss represents an essentially stochastic process. MC codes for muon transportation have to use some simplifications to optimize equilibrium between reasonable computation time, desirable accuracy and necessary statistics.

The usual approximation when calculating the muon propagation through a medium is the following: muon interactions with comparatively large energy transfers, i.e., when fraction of energy lost  $v = \Delta E/E$  exceeds some value  $v_{cut} \sim 10^{-3}-10^{-2}$ , are taken into account by direct simulation of  $v \geq v_{cut}$  for each single interaction according to shape of differential cross sections  $d\sigma(E, v)/dv$  (these interactions lead to “stochastic” energy loss or SEL) while the part of interaction with relatively small  $v$  is treated by the approximate concept of “continuum” energy loss (CEL), i.e., using the function  $[dE(E)/dx]_{CEL}$  which is obtained from formula

$$\begin{aligned} \left[ \frac{dE}{dx}(E) \right]_{CEL} = & \frac{N_A}{A} E \left( \int_{v_{min}^{\gamma}}^{v_{cut}} \frac{d\sigma^{\gamma}(E, v)}{dv} v dv \right) + \frac{N_A}{A} E \left( \int_{v_{min}^{e^+e^-}}^{v_{cut}} \frac{d\sigma^{e^+e^-}(E, v)}{dv} v dv \right) \\ & + \frac{N_A}{A} E \left( \int_{v_{min}^{pn}}^{v_{cut}} \frac{d\sigma^{pn}(E, v)}{dv} v dv \right) + \left[ \frac{dE}{dx}(E) \right]_{B-B} - \frac{N_A}{A} E \left( \int_{v_{cut}}^{v_{max}^{\delta}} \frac{d\sigma^{\delta}(E, v)}{dv} v dv \right). \end{aligned} \quad (1.1)$$

Here  $N_A$  is the Avogadro number;  $A$  is the atomic weight; indexes  $\gamma$ ,  $e^+e^-$ ,  $pn$  and  $\delta$  correspond to bremsstrahlung, direct  $e^+e^-$ -pair production, photonuclear interaction and knock-on electron production, respectively;  $v_{min}$  and  $v_{max}$  are the minimum and the maximum kinematically allowed fraction of energy lost for corresponding process; term  $[dE(E)/dx]_{B-B}$  represents Bethe-Bloch formula with correction for density effect. Notice that actually one has to compute CEL, as well as all total cross sections separately for each kind of atoms given material consists of and then add them to each other with different weights but here and below in the text we omit this detail and give only general expressions. One is forced to decompose energy loss because simulation of all interactions with  $v \geq v_{min}$  would result in infinite computation time due to steep dependence of muon cross sections on  $v$  (they decrease with  $v$  at least as  $d\sigma(E, v)/dv \propto v^{-1}$  and for some processes are not finite at  $v \rightarrow 0$ ). Number of interactions to be simulated per unit of muon path grows, roughly, as  $N_{int} \propto v_{cut}^{-1}$  along with computation time. On the other hand, setting  $v_{cut}$  to large value may affect simulation accuracy. Thus, the question is *how large value of  $v_{cut}$  may be chosen to keep result within desirable accuracy?*

The second problem can be formulated as follows: *if it is necessary to include ionization in SEL, at all?* Small energy transfers strongly dominate at knock-on electron production ( $d\sigma^{\delta}(E, v)/dv \propto v^{-2}$ ), so this process is almost non-stochastic and it seems to be reasonable to omit the last term in expression for CEL (Eq. (1.1)) and exclude knock-on electrons from simulation procedure when simulating SEL, i.e., to treat ionization completely as “continuous” process which saves computation time noticeable. How much does it affect the result of simulation?

Influence of these factors on simulated result was discussed in literature (see, e.g., Refs. [1–4]) but, in our opinion, more detailed analysis on which further improvement of muon propagation MC algorithms could be based is still lacking. In presented work we undertook an attempt to investigate in details with the MUM (MUons+Medium) muon propagation code (Ref. [5]) the influence of  $v_{cut}$  and model of ionization energy loss upon result of MC simulation. Resulting errors are compared with uncertainties which come from parametrization accuracy both for sea-level muon spectrum and for muon cross sections. Our analysis allows to rank different uncertainties according to their importance, expose ones to which special attention should be put and choose the most adequate setting of MC parameters at simulation for this or that purpose.

We briefly describe the main features of the MUM code which was a basic instrument for reported investigation and give a short review of principal variables have been studied in Sec. II. Sec. III presents results of simulation for survival probabilities and muon flux intensities with different settings of simulation parameters. In Sec. IV we analyse results of simulations presented in Sec. III, compare them with other published results and give some basic inferences. Sec. V gives general conclusions.

## II. METHOD

Description of the MUM code which was used to obtain presented results has been published in Ref. [5]. Here we only dwell briefly on those features of the code which seem to be important in the frame of discussed problem:

- the most recent results on parametrizations for muon cross sections are used in the code;
- we tried to decrease the “methodical” part of systematic error due to interpolation, numerical integration, etc., down to as low level as possible, especial attention was put on simulation procedures for free path and fraction of energy lost;
- the most important parameters, like value of  $v_{cut}$ , model for ionization loss, kind of medium, parametrizations for muon cross sections are changeable and represent input parameters for initiation routine;

The accuracy of simulation algorithm used in MUM was shown in Ref. [5] to be high enough to perform systematically significant analysis which is presented below.

In order to study how different factors influence upon result of simulation we performed several sets of simulations both for propagation of monoenergetic muon beam and muons sampled by real sea-level spectrum (in the later case we limited ourselves by simulation only vertical muons) through pure water down to depths from  $D = 1$  km to  $D = 40$  km. Of course, depths of more than several kilometers of water for vertical muons do not concern any real detector but simulations for larger depths allow us to study general appropriatenesses for large muon ranges which correspond, for instance, to nearly horizontal muons. Several runs were done for standard rock ( $A = 22$ ,  $Z = 11$ ,  $\rho = 2.65$  g cm $^{-3}$ ), as well. Muons whose energy decreased down to  $E = 0.16$  GeV (the Cherenkov threshold for muon in water) were considered as stopped ones. We tested different settings of parameters which were as follows.

- (a)  $v_{cut}$ , which changed within a range of  $v_{cut} = 10^{-4}$  to  $v_{cut} = 0.2$ . Actually, “inner” accuracy of the MUM code becomes somewhat worse at  $v_{cut} > 5 \times 10^{-2}$ , especially if fluctuations in ionization are not simulated (Ref. [5]). So, results for  $v_{cut} = 0.1$  for  $v_{cut} = 0.2$  are presented here only to illustrate some general qualitative appropriatenesses.
- (b) Model for ionization loss which was treated both as completely “continuous” and “stochastic” with simulation of energy lost if knock-on electron energy  $\Delta E \geq v_{cut} E$ .
- (c) Parametrization for vertical sea-level conventional atmospheric muon spectrum. Two spectra were tested, namely one proposed in Ref. [6] (basic):

$$\frac{dN}{dE} = \frac{0.175 E^{-2.72}}{cm^2 s sr GeV} \left( \frac{1}{1 + \frac{E}{103 GeV}} + \frac{0.037}{1 + \frac{E}{810 GeV}} \right), \quad (2.1)$$

and widely used Gaisser spectrum (Ref. [7]):

$$\frac{dN}{dE} = \frac{0.14 E^{-2.7}}{cm^2 s sr GeV} \left( \frac{1}{1 + \frac{E}{104.6 GeV}} + \frac{0.054}{1 + \frac{E}{772.7 GeV}} \right). \quad (2.2)$$

- (d) Parametrization for total cross section for absorbtion of a real photon of energy  $\nu = s/2m_N = vE$  by a nucleon at photonuclear interaction which was treated both according to Bezrukov-Bugaev parametrization proposed in Ref [8] (basic):

$$\sigma_{\gamma N} = [114.3 + 1.647 \ln^2(0.0213 \nu)] \mu b \quad (2.3)$$

and ZEUS parametrization (Ref. [9]):

$$\sigma_{\gamma N} = (63.5 s^{0.097} + 145 s^{-0.5}) \mu b \quad (2.4)$$

( $\nu$  and  $s$  in GeV and  $GeV^2$ , correspondingly).

- (e) A factor  $k_\sigma$  which all muon cross sections were multiplied by to test influence of uncertainties in cross sections parametrization (and, concequently, in energy loss) upon result. We used  $k_\sigma = 1.0$  as a basic value but in some cases set also  $k_\sigma = 0.99$  and  $k_\sigma = 1.01$ , which corresponds to decrease and increase of total energy loss by 1%, respectively. Note that it is an “optimistic” evaluation, the real accuracy of existing parametrization for muon cross sections is worse (see Refs. [10,11]).

For each run we fixed the muon spectra at final and several interim depths. The differences between obtained spectra were a point of investigation.

### III. RESULTS

#### A. Propagation of monoenergetic muon beams

At the first set of simulations we propagated monoenergetic muon beams of 4 fixed initial energies  $E_s = 1$  TeV, 10 TeV, 100 TeV and 10 PeV down to depths  $D = 3.2$  km, 12 km, 23 km and 40 km, respectively, through pure water. In each case propagation of  $10^6$  muons was simulated. Fig. 1 shows resulting survival probabilities  $p = N_D/N_s$  (where  $N_s = 10^6$  is initial number of muons and  $N_D$  is number of muons which have survived after propagation down to depth  $D$ ) vs.  $v_{cut}$  for final and five interim depths. Two curves are given on each plot for two models of ionization. Also results for  $k_\sigma = 1.00 \pm 0.01$  and for parametrization (2.4) are presented as simulated with the most accurate value  $v_{cut} = 10^{-4}$ .

The following appropriatenesses are visible.

- (a) In most cases with the exception of some plots of the lower row and the left column in Fig. 1 (which corresponds to low survival probabilities and low muon initial energies, respectively) uncertainty in our knowlege of muon cross sections gives the principal effect which essentially exceeds ones from other tested parameters.
- (b) The difference between survival probabilities for two models of ionization is the less appreciable the larger muon energy is. It is quite understandable because at muon energies  $E < 1$  TeV ionization represents the great bulk of total energy loss, and vice versa, it becomes minor at  $E > 1$  TeV. Thus, contribution which is given by ionization at higher energies is small and, the more, its fluctuations do not play an important role. For muons with initial energies  $E \gg 1$  TeV fluctuations in ionization become important only at very last part of muon path and “are not in time” to produce some noticeable effect.
- (c) Generally, parametrizations (2.3) and (2.4) do not show a noticeable difference in survival probabilities, in most cases it is within statistical error or exceeds it only slightly.
- (d) Increase of  $v_{cut}$  gives effect of both signs in survival probabilities: function  $p(v_{cut})$  grows at the beginning of muon path and falls at the last part. The same “both-sign” dependencies are observed for ionization model.

- (e) For  $v_{cut} \leq 0.02 - 0.05$  there is almost no dependence of survival probability on  $v_{cut}$  with the exception of very last part of muon path where survival probability becomes small. Generally, dependence  $p(v_{cut})$  is the less strong the larger initial muon energy is.

The last item is illustrated complementary by Fig. 2 and Fig. 3 which show that for all initial energies  $E_s$  simulated survival probability does not depend, in fact, on  $v_{cut}$  until 90% (for  $E_s = 1$  TeV) to 99.5% (for  $E_s = 10$  PeV) muons have been stopped.

It was shown above *what is result* of simulations with different models of ionization and values of  $v_{cut}$ . It was a special point of interest for us to track *how does it* influence upon behaviour of survival probability.

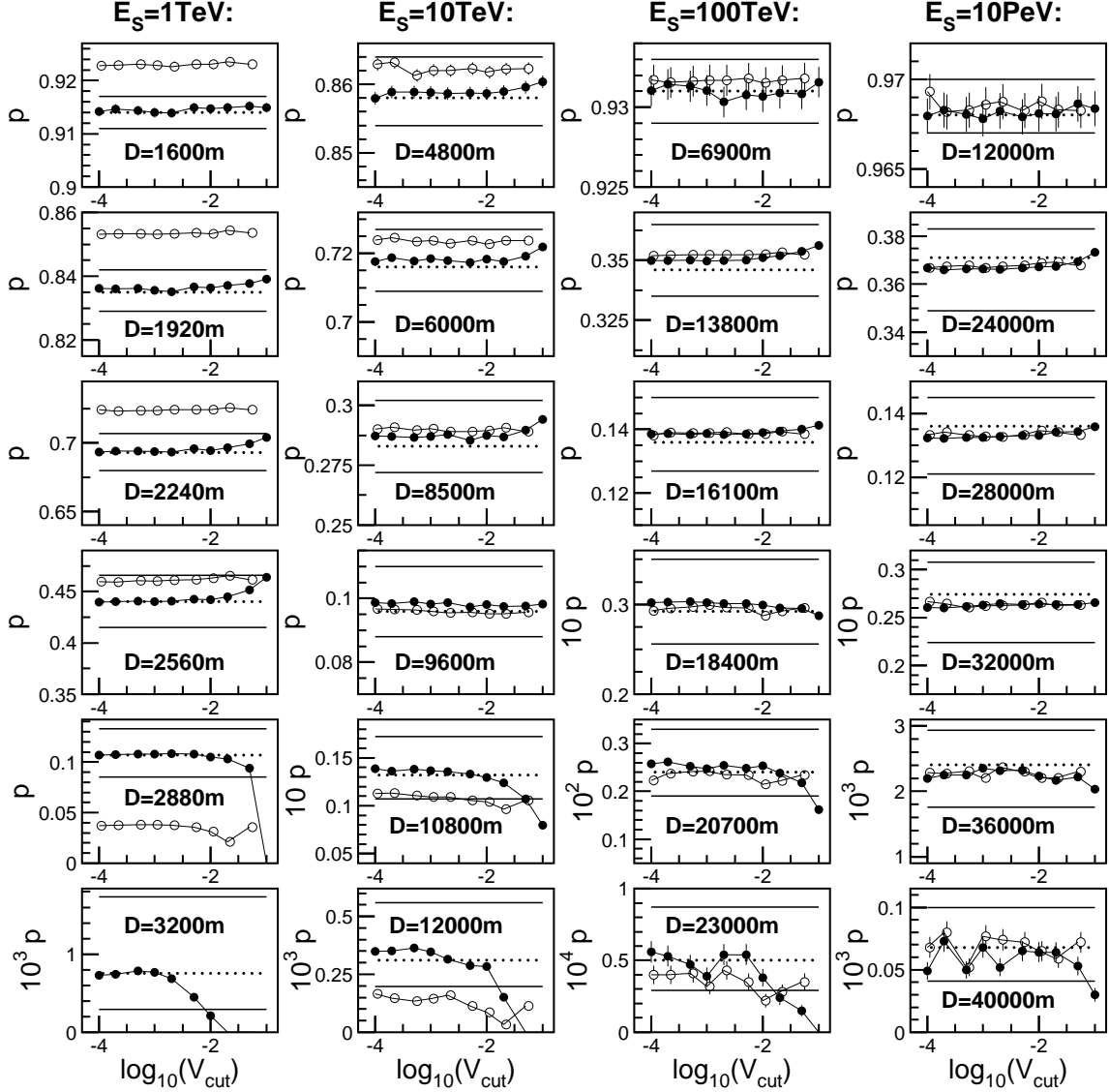


FIG. 1. Survival probabilities  $p = N_D/N_s$  (where  $N_s = 10^6$  is initial number of muons and  $N_D$  is number of muons which have survived after propagation down to depth  $D$  in pure water) vs.  $v_{cut}$ . Values of  $p$  were obtained as a result of MC simulation for monoenergetic muon beams with initial energies  $E_s = 1$  TeV (1st column of plots), 10 TeV (2nd column), 100 TeV (3rd column) and 10 PeV (4th column). Each column contains six plots which correspond to six depths  $D$  (which differs for different  $E_s$ ). Closed circles represent survival probabilities which were simulated with ionization energy loss included in SEL along with other types of muon interactions. Open circles correspond to computation with completely “continuous” ionization. Two horizontal solid lines on each plot show the value for survival probability computed with all muon cross sections multiplied by a factor  $k_\sigma = 1.01$  (lower line) and  $k_\sigma = 0.99$  (upper line) for  $v_{cut} = 10^{-4}$ . Horizontal dotted lines correspond to  $v_{cut} = 10^{-4}$  and cross section for absorbtion of a real photon at photonuclear interaction parametrized according to recent ZEUS data (Ref. [9], Eq. (2.4)) instead of parametrization proposed by L.B. Bezrukov and E.V. Bugaev (Ref. [8], Eq. (2.3)) which is basic in the MUM code. Note different scales at Y-axis.

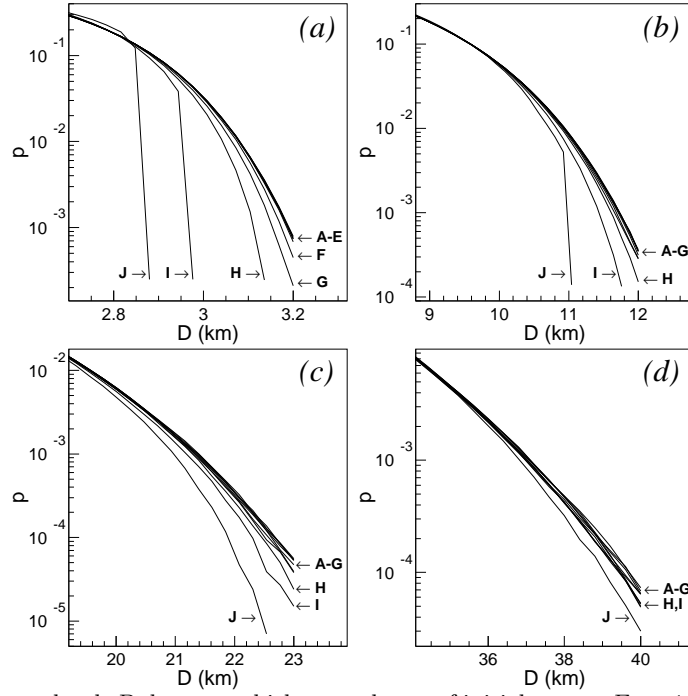


FIG. 2. Survival probability  $p$  vs. depth  $D$  down to which muon beam of initial energy  $E_s = 1$  TeV (a), 10 TeV (b), 100 TeV (c) and 10 PeV (d) propagates through pure water. On each plot 10 lettered curves which correspond to different values of  $v_{cut}$  are shown. Meaning of letters is as follows: A -  $v_{cut} = 10^{-4}$ , B -  $v_{cut} = 2 \times 10^{-4}$ , C -  $v_{cut} = 5 \times 10^{-4}$ , D -  $v_{cut} = 10^{-3}$ , E -  $v_{cut} = 2 \times 10^{-3}$ , F -  $v_{cut} = 5 \times 10^{-3}$ , G -  $v_{cut} = 10^{-2}$ , H -  $v_{cut} = 2 \times 10^{-2}$ , I -  $v_{cut} = 5 \times 10^{-2}$ , J -  $v_{cut} = 10^{-1}$ . This figure displays results were obtained by simulation with ionization loss included in SEL. Statistical errors which cause some unsmoothness of curves at small  $p$  are not shown. Dependence  $p$  upon  $v_{cut}$  becomes noticeable only at the last  $\approx 1/8$  of muon beam path where the majority of muons (from 90% to 99.5%, depending upon  $E_s$ ) has been stopped.

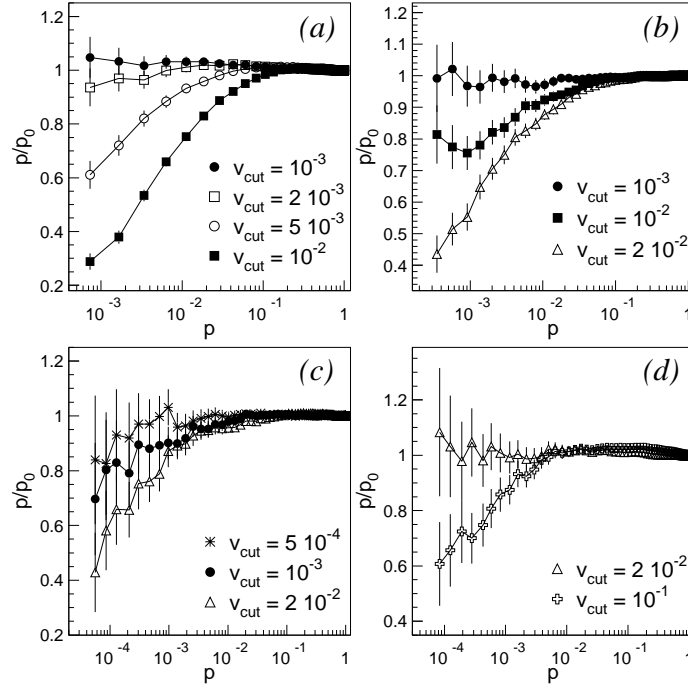


FIG. 3. Relation  $p/p_0$  vs.  $p$ .  $p$  is survival probability for muon beam of initial energy  $E_s = 1$  TeV (a), 10 TeV (b), 100 TeV (c) and 10 PeV (d) at propagation through pure water with ionization included in SEL as simulated for different values of  $v_{cut}$ ;  $p_0$  is survival probability simulated under the same conditions for  $v_{cut} = 10^{-4}$ . Difference in  $p/p_0$  becomes noticeable only at small values of  $p$ , i.e., at the last part of muon beam path.

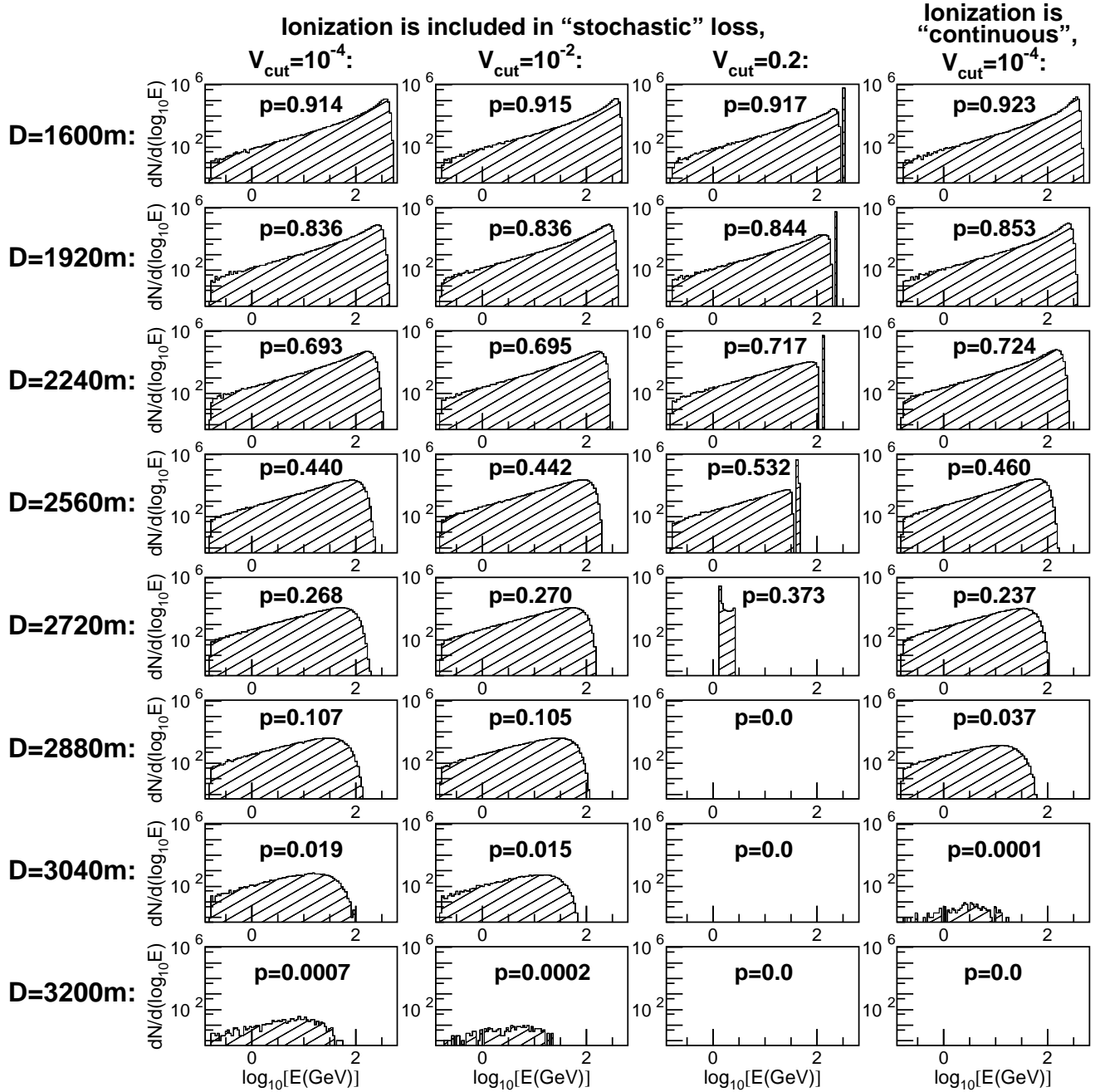


FIG. 4. Muon spectra at different depths  $D$  in pure water resulting from  $10^6$  muons with initial energy  $E_s = 1$  TeV obtained by simulation with four models. The first three columns represent spectra obtained with ionization included in SEL for  $v_{cut} = 10^{-4}$  (1st column),  $10^{-2}$  (2nd column), and 0.2 (3rd column). 4th column contains spectra obtained for entirely “continuous” ionization and  $v_{cut} = 10^{-4}$ . On each plot value of survival probability  $p$  is indicated (without statistical error which is negligible for the most plots and, on desire, can be easily calculated taking into account that  $p = N_D/10^6$ , where  $N_D$  is number of muons which have survived after propagation down to depth  $D$ ).

Fig. 4 shows how muon spectrum resulting from monoenergetic muon beam with initial energy  $E_s = 1$  TeV transforms when propagating through pure water down to the depth of 3.2 km. Simulated results for four settings of parameters are presented by four columns of plots. The first three columns represent spectra obtained with ionization included in SEL for  $v_{cut} = 10^{-4}$ ,  $10^{-2}$  and 0.2. 4th column contains spectra simulated with entirely “continuous” ionization and  $v_{cut} = 10^{-4}$ . The spectra grouped into the first column represent some “ethalon” because they were simulated with the most accurate tuning both for  $v_{cut}$  and ionization model. The first three columns demonstrate that “compactness” of spectra at the same depth is the higher the more value of  $v_{cut}$  is. Put your attention to the right edge of spectra which shifts toward low energies when  $v_{cut}$  increases (it is the most noticeably for  $v_{cut} = 0.2$ ). The reason is that

at any depth energy of the most energetic muons in simulated beam is determined by CEL. These muons due to statistical fluctuations did not undergo interactions with  $v \geq v_{cut}$  and, consequently, lost energy only by CEL which increases when  $v_{cut}$  increases. That is why the maximum energy in simulated muon beam is lower for large values of  $v_{cut}$ . Fraction of muons which did not undergo an “catastrophic” act with  $v \geq v_{cut}$  till given depth grows with increase of  $v_{cut}$  because free path between two sequential interactions with  $v \geq v_{cut}$  grows approximately as  $\bar{L} \propto v_{cut}$ . It leads, in particular, to distinctly visible picks in spectra for  $v_{cut} = 0.2$  consisted just of muons which lost energy only by CEL. Also, some deficit of low energy muons appears if one sets  $v_{cut}$  to a large value. In this case left edge of spectrum is provided only with muons which interacted with large fraction of energy lost while for smaller  $v_{cut}$  an additional fraction of muons comes here. As a result simulated spectrum of initial monoenergetic muons at given depth is more narrow if  $v_{cut}$  is large and, on the contrary, more wide if  $v_{cut}$  is small.

Now it is easy to understand how value of  $v_{cut}$  influences on simulated survival probabilities. When simulated muon beam goes through medium losing energy both in CEL and SEL processes, its spectrum is constantly shifting to the left (energy decreases). For  $v_{cut} = 10^{-4}$  the left part of spectrum reaches  $E = 0$  at a smaller depth comparing with larger  $v_{cut}$  (because in this case spectrum is wider) and survival probability starts to decrease. At the same depth survival probability for  $v_{cut} = 10^{-2}$  and  $v_{cut} = 0.2$  is still equal to 1. Thus, for the first part of path the survival probability is always larger for large  $v_{cut}$ . At some depth (which is equal to approximately 2.8 km for considered case) compactness of spectra simulated with large  $v_{cut}$  starts to play an opposite role. Due to this compactness and higher CEL muons stop faster comparing with accurate simulation. So, at the final part of the beam path simulated survival probability for large  $v_{cut}$  decreases faster comparing with accurate simulation and, for instance, for  $v_{cut} = 0.02$  the rest of muon beam which reaches the depth of  $D = 2.72$  km (37% of initial number of muons) completely vanishes within the next 30 m of path, while some fraction of muons simulated with  $v_{cut} = 10^{-4}$  (0.07%) escapes down to the depth of  $D = 3.2$  km.

Qualitatively the same effect leads to the same results if one treats ionization as completely “continuous” energy loss. Again, spectra becomes more narrow since fluctuations in ionization do not work and, as a consequence, survival probability becomes significantly higher comparing with simulation with accurate treatment of ionization at the beginning of muon beam path and falls down essentially faster at the final part of path.

Results presented in Sec. III A show the significant influence which both model of ionization and value of  $v_{cut}$  have over survival probability for monoenergetic muon beam. But for practical purposes the more important is *how this factors do work for real atmospheric muons with a power spectrum?*

## B. Propagation of muons sampled according to a power sea-level spectrum

In Fig. 5 we present intensity of vertical atmospheric muon flux  $I$  at different depths of pure water  $D$  from 1 km to 20 km vs.  $v_{cut}$  as simulated with muons sampled according to sea-level spectrum (2.1). Simulation continued until  $10^4$  muons reached given depth. Curves for two models of ionization are shown for each depth along with results for  $k_\sigma = 1.00 \pm 0.01$  at  $v_{cut} = 10^{-4}$ , parametrization (2.4) at  $v_{cut} = 10^{-4}$ , sea-level muon spectrum (2.2) at  $v_{cut} = 10^{-4}$  and all energy loss treated entirely as CEL (for depths  $D \leq 5$  km only).

General appropriatenesses for real muon spectrum are qualitatively the same as observed for monoenergetic muon beams.

- (a) For all depths at which neutrino telescopes are located it was found to be better to take into account fluctuations of energy loss simulated by *any* model than to treat energy loss as completely “continuous”: muon flux intensity computed with non-stochastical model of energy loss is always less comparing with stochastical model, the difference reaches 10 % at 3 km w.e. and 40% at 5 km w.e. (at the depth of 20 km of pure water vertical muon flux computed with ignorance of fluctuations is only 10 % of simulated flux).
- (b) Like in a case for monoenergetic beams 1%-uncertainty in muon cross sections plays the principal role for resulting error in simulated muon depth intensity. This error has a tendency to grow with depth from  $\pm 2.5\%$  at depth of 1 km w.e. to  $\pm 15\%$  at 20 km w.e.. But a particular case of this uncertainty, namely difference between parametrizations for total cross section for absorption of a real photon by a nucleon at photonuclear interaction from Refs. [8,9], does not lead to a significant difference in resulting intensity.
- (c) Difference between muon spectra (2.1) and (2.2) leads to uncertainty from -4% ( $D = 1$  km) to 16% ( $D = 20$  km).
- (d) Error which appears due to simplified, entirely “continuous” ionization lies, commonly, at the level of 2–3 %.

- (e) Dependence of simulated muon flux intensity upon  $v_{cut}$  is the most weak one comparing with other studied error sources. Function  $I(v_{cut})$  is almost a constant if  $v_{cut} \leq 0.02$ – $0.05$  and changes in a range  $\pm 1$ – $2\%$  which is very close to statistical error.

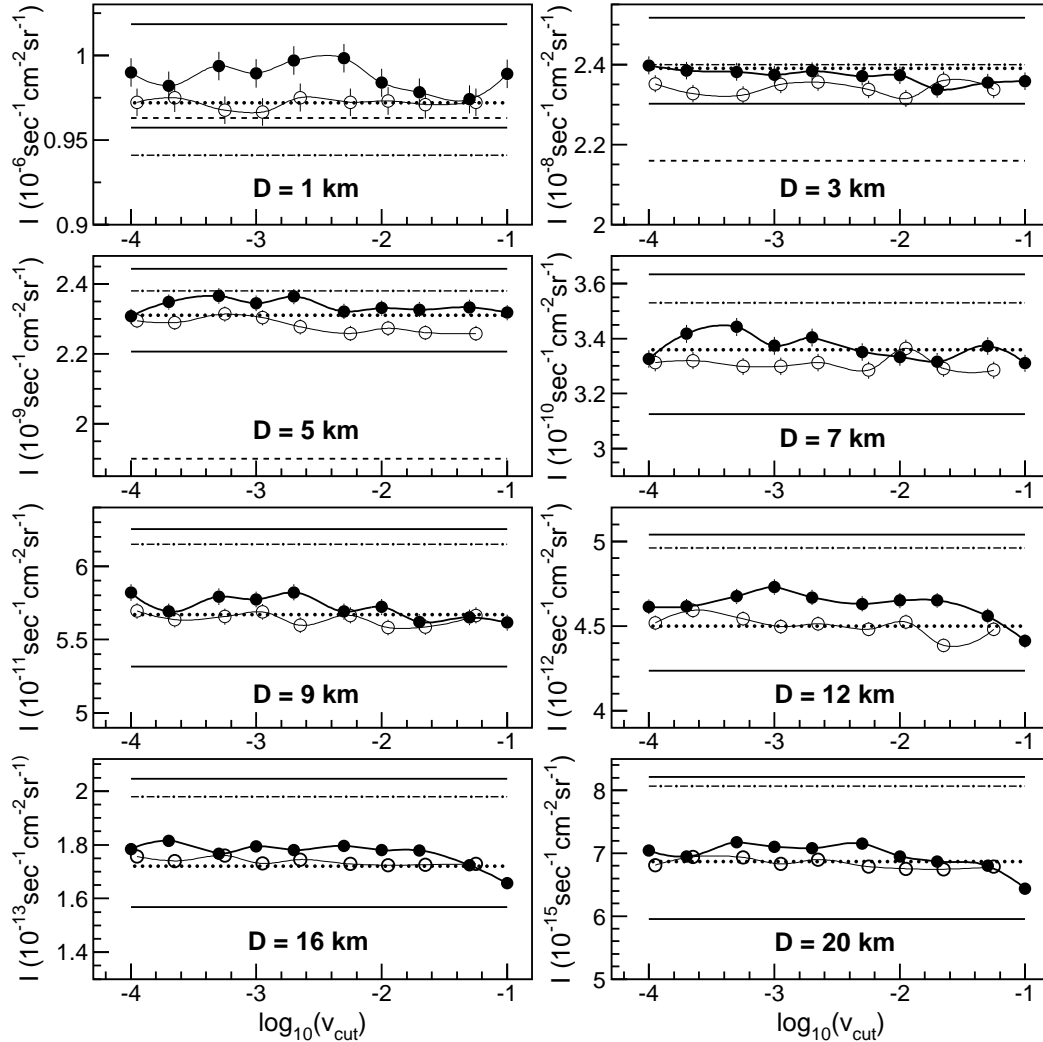


FIG. 5. Intensity of vertical atmospheric muon flux  $I$  at different depths  $D$  of pure water vs.  $v_{cut}$  as obtained by simulation with muons sampled according to sea-level spectrum from Ref. [6] (Eq. (2.1)). Closed circles: ionization is included in SEL; open circles: ionization is completely “continuous”. Two horizontal solid lines on each plot show value for survival probability simulated with all muon cross sections multiplied by a factor  $k_\sigma = 1.01$  (lower line) and  $k_\sigma = 0.99$  (upper line) for  $v_{cut} = 10^{-4}$ . Dashed lines on plots for  $D \leq 5$  km correspond to intensity which was calculated for all energy loss treated as “continuous”. Dash-dotted lines show intensity of vertical muon flux simulated with ionization included in SEL,  $v_{cut} = 10^{-4}$  and muons sampled according to Gaisser sea level spectrum (Ref. [7], Eq. (2.2)). Horizontal dotted lines correspond to  $v_{cut} = 10^{-4}$  and cross section for absorption of a real photon at photonuclear interaction parametrized according to Ref. [9] (Eq. (2.4)) instead of parametrization proposed in Ref. [8] (Eq. (2.3)) which is basic in the MUM code.

We also tried to reveal how value of  $v_{cut}$  and model for ionization influence upon differential muon depth spectra. No differences were detected which would exceed statistical error. It is illustrated by two figures. Fig. 6 displays simulated mean energies for vertical muon flux at different depths  $D$  of pure water vs.  $v_{cut}$  as simulated with muons sampled according to sea-level spectrum (2.1). Two dependencies are presented at each depth for two model of ionization loss. No appropriatenesses are visible on the plot. In Fig. 7 we present simulated differential muon spectra at four depths  $D = 1$  km, 5 km, 10 km, 20 km. Two spectra are displayed on each plot obtained *i)* for the most accurate value of  $v_{cut} = 10^{-4}$  and *ii)* for the most rough case  $v_{cut} = 0.2$ . Spectra are normalized to  $10^4$  muons and at each depth are divided into three parts along muon energy to keep linear scale at Y-axis. Again, no statistically significant differences are marked.



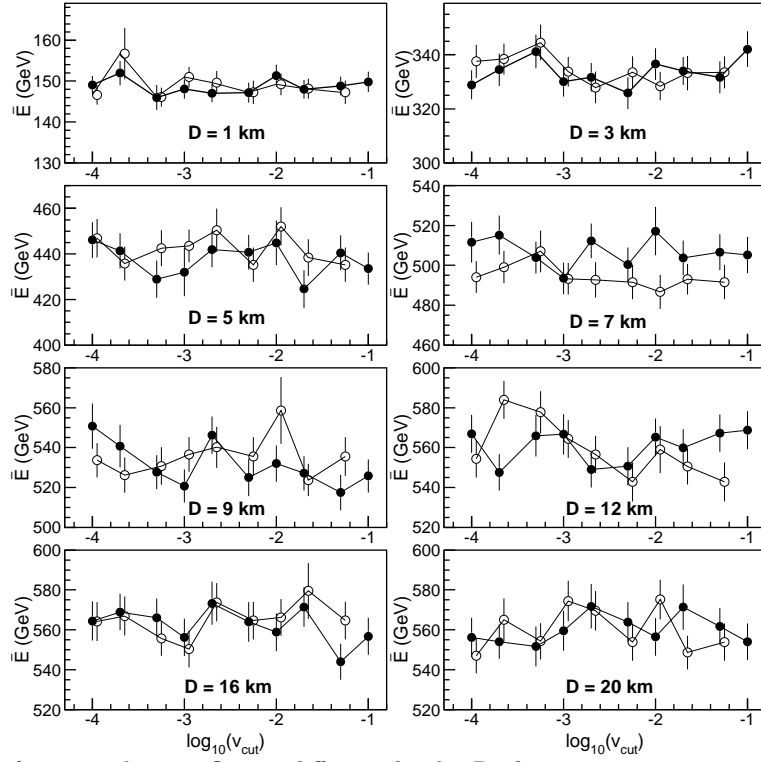


FIG. 6. Mean energy for vertical muon flux at different depths  $D$  of pure water vs.  $v_{cut}$  as obtained by simulation with muons sampled according to sea-level spectrum from Ref. [6] (Eq. (2.1)). Closed circles: ionization is included in SEL; open circles: ionization is completely “continuous”.

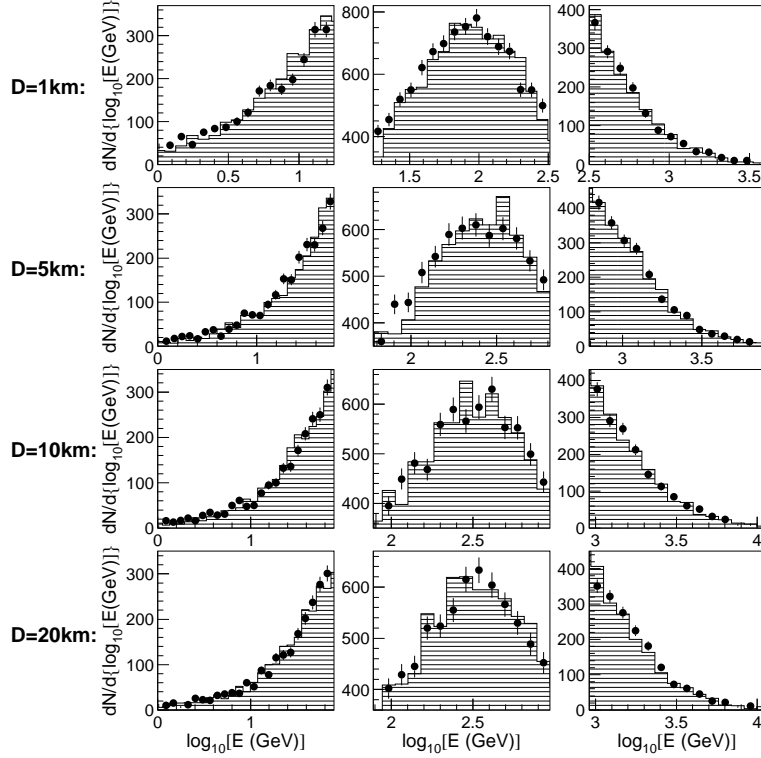


FIG. 7. Simulated differential muon spectra at four depths  $D = 1$  km, 5 km, 10 km, 20 km of pure water. Two spectra are displayed on each plot, namely simulated for  $v_{cut} = 10^{-4}$  (histogram) and  $v_{cut} = 0.2$  (closed circles). Spectra are normalized for  $10^4$  muons with energies  $E > 0.16$  GeV and at each depth are divided into three parts along muon energy to keep linear scale at Y-axis.

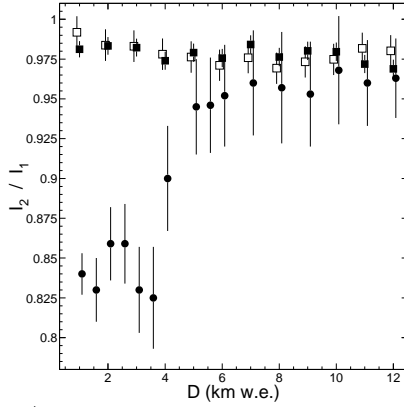


FIG. 8. Dependencies for relation  $I_2/I_1$  vs. water equivalent depth in standard rock as computed in this work (closed squares), in Ref. [3] (open squares) and in Ref. [4] (closed circles).  $I_1$  is depth intensity for vertical muon flux simulated with ionization included in SEL,  $I_2$  is one simulated with entirely “continuous” ionization. Data for this work are obtained for sea-level spectrum from Ref. [6] (Eq. (2.1)) and  $v_{cut} = 10^{-3}$ ; data from Ref. [3] represent result of simulation for spectrum from Ref. [7] (Eq. (2.2)) and  $v_{cut} = 10^{-3}$ ; data from Ref. [4] were simulated with spectrum from Ref. [12] with “small transfer grouping” technique.

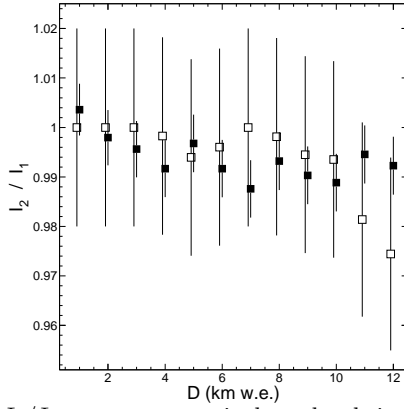


FIG. 9. Dependencies for relation  $I_2/I_1$  vs. water equivalent depth in standard rock as computed in this work (closed squares) and in Ref. [3] (open squares).  $I_1$  is depth intensity for vertical muon flux simulated with entirely “continuous” ionization and  $v_{cut} = 10^{-3}$ ,  $I_2$  is one simulated with the same treatment of ionization and  $v_{cut} = 10^{-2}$ . Data for this work are obtained for sea-level spectrum from Ref. [6] (Eq. (2.1)); data from Ref. [3] represent result of simulation for spectrum from Ref. [7] (Eq. (2.2)).

#### IV. DISCUSSION

Results obtained in this work are evidence of accuracy in parametrizations for muon cross sections and spectra to be the principal source of uncertainties when simulating muon flux at depths where neutrino telescopes are located. Both for monoenergetic muons and muons sampled according to a power spectrum it gives uncertainty at least of 2–4% to 10–15% in resulting intensity of muon flux, depending upon depth. Unfortunately, this level has at present to be considered as a limit for accuracy of muon propagation codes. Influence of model for ionization exceeds this limit only for monoenergetic muons with sea-level muon energies  $E \leq 10$  TeV and only if level of observation is at very last stage of muon path where major fraction of initial muon energy has been lost. Actually, due to steep shape of atmospheric muon power spectrum, an essential part of muons reaches detector location being just on the last part of path. Therefore effect remains noticeable also for real atmospheric muons but in this case uncertainty was found in this work to be much less: 2–3%, which is in an excellent agreement with Refs. [1,3], while Ref. [4] predicts much more significant difference (up to 17%). We suppose this disagreement may result from the fact that “small transfer grouping” technique used for simulation in Ref. [4] treats muon cross sections to be constant between two interactions in contrast with algorithm used in the MUM code. As was shown in Ref. [5], in this case switching off the fluctuations in ionization leads to an additional amplification in effective energy loss for muon energies  $E \leq 1$  TeV and, consequently, simulated muon flux intensity must decrease at relatively small depths where muon spectrum is

formed by muons with sea-level energies just in a range  $E_s \sim 10^2\text{--}10^3$  GeV. In Ref. [3] the same simplification was used but reported results were obtained by simulation with  $v_{cut} = 10^{-3}$ . At this value of  $v_{cut}$  role of correct treatment for free path is not significant. Choice of value for  $v_{cut}$  is of even less importance and again, it is more critical if one investigates monoenergetic muon beam but for power spectrum alteration in  $v_{cut}$  within  $v_{cut} \leq 0.02\text{--}0.05$  leads only to 1–2% differences in simulated muon flux intensities which, again, is in a good agreement with level of errors reported in Ref. [3]. Differences between muon flux intensities simulated for different models of ionization and values of  $v_{cut}$ , as obtained in given work and in Refs. [4,3], are presented in Fig. 8 and Fig. 9.

Since computation time which is necessary for simulation depends strongly upon used model of ionization loss and upon  $v_{cut}$ , it seems to be reasonable for most purposes to set  $v_{cut}$  to  $v_{cut} \approx 0.01\text{--}0.05$ . Anyway, in this case resulting error will be less comparing with error caused by accuracy of parametrization for muon cross sections and sea-level muon spectrum, if one treats free path  $L$  by an accurate method (Ref. [5]). Note that “inner” accuracy of the MUM code was found to be worse than “outer” one (which results from uncertainties of muon cross sections) if ionization is set to “continuous” model and  $v_{cut} > 10^{-2}$ . For fluctuated ionization inner accuracy remains high enough at least till  $v_{cut} = 0.05$  (Ref. [5]). So, using MUM one should set  $v_{cut}$  to  $v_{cut} \leq 0.01$  if ionization is entirely continuous and to  $v_{cut} \leq 0.05$  for ionization included in SEL. For test runs it is possible to set  $v_{cut}$  even to larger values. On the other hand for some methodical purposes it may be useful to simulate fluctuations in knock-on electron production and choose more fine  $v_{cut}$ , e.g., if one wants to exclude an additional error when comparing results of simulations for different models of muon sea-level spectrum with each other.

It is impossible to consider all particular cases and give some strict conformity between setting of parameters at muon MC propagation code and problem to be solved. We had for an object to create a code which would allow to change easy the most important parameters and to perform an analysis which would allow anyone to choose these parameters according to one’s purposes and taste. We hope this object has been achieved with this article.

## V. CONCLUSIONS

We have presented the detailed investigation for dependence of computed depth muon flux upon the most important parameters of muon propagation MC algorithm, which was done with the MUM (MUons+Medium) code (Ref. [5]). Contributions of different simplifications and uncertainties to the resulting error have been analysed and ranked. We hope that presented work can be useful for further development of muon transport algorithms which are necessary for adequate analysis of muon data obtained at underground/water/ice neutrino detectors.

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